
Théo Vischel, Thierry Lebel *

Laboratoire d'étude des Transferts en Hydrologie et Environnement (UMR 5564), IRD, BP 53, F-38041 Grenoble cedex 9, France

Received 26 October 2005; received in revised form 8 September 2006; accepted 11 September 2006

Abstract As in many other semi-arid regions in the world, the Sahelian hydrological environment is characterized by a mosaic of small endoreic catchments with dry soil surface conditions producing mostly Hortonian runoff. Using an SCS-type event based rainfall–runoff model, an idealized modeling experiment of a Sahelian environment is set up to study the sensitivity of runoff to small scale rainfall variability. A set of 548 observed rain events is used to force the hydrological model to study the sensitivity of runoff to the time and space variability of rainfall input. The rainfall time variability sensitivity analysis shows that preserving the event rain depth without representing the main variabilities of the hyetograph intensities can translate into a runoff error of 65% in the worst case. On a virtual mosaic of 1-km² catchments covering 10,000 km², the simulated runoff shows a high sensitivity to a decrease of the spatial resolution of event rain fields from 1×1 km² to 100×100 km². For the catchments characterized by low runoff coefficients, which are the most sensitive to rainfall variability, at the coarsest spatial resolution of 100×100 km², the global runoff computed from the 548 events is underestimated by 50% with respect to the runoff simulated from the 1×1 km² resolution rain fields. The threshold resolution of 20 km was identified as a characteristic spatial scale, over which the performance of the model rapidly decreases. Looking at the influence of the number of available rain gauges, the effect of spatial aggregation depends on the density of the rain gauge network with lower effect for sparser networks.

© 2006 Elsevier B.V. All rights reserved.

KEYWORDS Rainfall space–time resolution; Runoff; SCS-type hydrologic model; Semi-arid endoreic environment

* Corresponding author.
E-mail address: Lebel@hmg.inpg.fr (T. Lebel).

0022-1694/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.jhydrol.2006.09.007

Introduction

The contradictory impacts of the West African drought on runoff

As shown by Lebel et al. (2003), the drought that has affected West Africa since the end of the 1960s has produced a severe decrease of the river flows of the large Sahelian hydrological systems (Niger, Senegal). In contrast to this large scale river flow deficit, small scale studies showed that, locally, runoff has in fact increased during the last thirty years, due to changes in the vegetation cover and consequently in the hydrodynamic soil surface properties (Seguis et al., 2004). Explaining such different behaviors depending on the scale considered is a real challenge. It must take into account the fact that the type of vegetation, the hydrological environment, and the history of land cover change, among other factors, vary significantly across the Sahel. Large scale river systems thus integrate a range of smaller catchments with different hydrological behaviors. Local modifications of the water balance are often as important to the populations as the fluctuations of the large river flows. This is especially the case in the Sahel where a major part of the region is composed of a multitude of small catchments (typical size between a few ha and a dozen km²) characterized by an endoreic behavior. Such behavior is mainly due to the siting up of the river channels, impeding the runoff to be drained out of the catchments. This implies that the runoff accumulates into pools which are the outlet of the catchments (see e.g. Desconnetes et al., 1997). From these pools, water evaporates and infiltrates into deep aquifers, but contributes very little to large scale river flows. The Nile, the Niger, and the Senegal rivers are all emblematic examples of surface water originating in upstream mountains and crossing the downstream regions with little local input.

With respect to a changing hydrology, it is important to consider the notion of adaptation. The prospect of the potentially devastating consequences of climate change on the Sahelian agriculture — and the societies themselves — has led to studying adaptation strategies (see e.g. Mkankam Kamga, 2001; Van Duivenbooden et al., 2002; Ben Mohamed et al., 2002; Butt et al., 2005). In such a context, it is puzzling that most studies dealing with the hydrological impact of climate variability in the tropics only consider the large scale aspect of the problem (see e.g. Arnell et al., 2003; Arora and Boer, 2001). The approach taken in these studies is to force water budget models by coarse resolution climate scenarios. It must be assessed whether not properly taking into account the scale related effects in these water budget models could lead to significantly biased conclusions. More generally, it is pivotal to assess how rainfall variability affects the local water balance and whether low resolution/large scale studies allow characterizing the water cycle at smaller scales.

The rainfall scale issue in hydrological modelling studies: a brief review of literature

The impact of rainfall variability on the water cycle has long been identified as an important issue when one seeks to close the water balance of a catchment or a region. When the impact of spatial variability of rainfall is of concern, the problem is considered from two angles that are thoroughly described in several studies (see e.g. Bloschli and Sivapalan, 1995): (i) the spatial sampling problem, (ii) the spatial resolution problem. The spatial sampling problem refers to point measurement of rainfall gauge networks which are often too sparsely distributed to correctly capture the spatial variability of rainfall. In the case of Sahelian rainfall, the reader can refer to the companion paper of Balme et al. (in press) (hereafter referred to as 806) for a detailed analysis of the biases introduced in assessing the mean spatial rainfall from networks of varying densities: The impact of rainfall sampling error on runoff was mainly addressed in the early literature of the 1970s and 1980s. By progressively decreasing the rain gauge network density, it can be assessed how under-sampling the spatial rainfall variability impacts the quality of rainfall—runoff modelling and the accuracy of runoff estimation. A typical study is the pioneering work of Dawdy and Bergmann (1969) on the Santa Anita Creek basin (25 km²). The authors concluded that a misrepresentation of the spatial variability of rainfall could lead to a peak discharge error of more than 20%. Several subsequent studies dealing with catchments of varying size and environmental conditions all led to the similar conclusion that under-sampling the input rain fields has a significant impact on runoff assessment (see e.g. Wilson et al., 1979; Troutman, 1983; Faures et al., 1995; Shah et al., 1996; Dong et al., 2005).

The spatial resolution problem refers to data integrated over surfaces, and may arise from different situations: (i) in climate change studies, climate models used to produce scenarios have a coarse resolution, even when run in a regional mode, providing rainfall scenarios averaged over at least a few ten thousand km², which are far from the smaller resolutions required by hydrological models (Gleick, 1986); (ii) in operational hydrology, satellite information presents an attractive alternative/complement to rain gauge information in regions where ground information is sparse, but satellite rainfall estimates are also obtained over grid meshes far larger than the scales of interest to hydrologists. More recent studies turned their attention to the stricto sensu resolution problem. On a 256 × 256 km² area, Finnerty et al. (1997) showed that runoff could decrease to zero when using coarse rain field resolutions for which the model was not calibrated. On the 150-km² Walnut Gulch watershed (Arizona), Michaud and Sorooshian (1994a) found that the peak flow could be underestimated by more than 50% when the rain fields at the initial resolution of 0.05 × 0.05 km² were aggregated to the coarser 4 × 4 km² resolution. More recently, Liang et al. (2004) tested, among other hydrological variables, the impact of rainfall spatial resolution on the quality of model calibration based on the runoff of the 1233-km² Blue River watershed. They found that the finer the resolution, the better the calibration. A critical scale of 1/8 degree (about 14 km) was identified: for lower resolutions, errors started to be significant. Some studies found, however, that the fine spatial representation of rain fields was not so important (see e.g. two studies on contrasted hydrological environments: Obled et al. (1994) on a 71-km² Mediterranean watershed; Booij (2002), on the 30,000-km² Meuse basin). To explain such
contradictory results, Obled et al. (1994) argue that catchments for which infiltration-excess (Hortonian) runoff mechanisms are preponderant are more sensitive to spatial variability of rainfall than catchments for which runoff is produced by saturation-excess runoff mechanisms. This was verified by Winchell et al. (1998) who showed on the 102-km² Timber Creek watershed that a model formulation based on infiltration-excess runoff mechanisms was more sensitive to spatial rainfall resolution than a model formulation based on saturation-excess runoff mechanisms. As pointed out by Michaud and Sorooshian (1994a), the semi-arid catchments are predominantly of Hortonian type and are thus highly sensitive to the spatial variability of rainfall.

The time variability issue was also the focus of a few studies testing the sensitivity of rainfall–runoff modelling to the time step of the forcing rainfall. On the 7.5-km² Ralston Creek catchment Krajewski et al. (1991) noted the considerable effect of decreasing temporal resolution compared to the effect of varying spatial sampling. Michaud and Sorooshian (1994a) found that aggregating the time resolution from 4-min to 1-h could lead to a bias of 77% in peak flow estimation. A different approach was taken by Lambronn and Stephenson (1987) who evaluated the impact on runoff of time structure of the storms, i.e. the distribution of rainfall intensities inside storms. By comparing the runoff values computed from three types of synthetic hyetographs having rectangular, triangular and bimodal triangular shape, they found significant differences in simulations and show that triangular, and in particular bimodal, profiles were more adequate to reproduce runoff peaks and volumes.

**Objective of the study**

The objective of this paper is to assess the importance of accounting for the small scale variability of rainfall in rainfall–runoff studies over a typical Sahelian region. The study presented here adds to previous works on three accounts: (i) it makes use of an extensive rainfall data base spanning a large array of rain event types (548 rain events) and annual climatologic conditions (1990-2002); (ii) it addresses all three major fields of investigation reviewed above, comparing the respective effects of time variability of rainfall, spatial under-sampling, and decreasing resolution of rain fields (even though the emphasis is on spatial resolution effects); (iii) it focuses on the annual time scale rather than on the event time scale, even though the modelling itself is carried out at the event scale.

After a presentation of the data and hydrological model used (section Studied area, data and hydrological model), the idealized approach taken is described in section Methodology: an idealized approach of the scale issue. Section Sensitivity to time variability and hyetograph modelling presents the results on the influence on runoff of the representation of the time structure of rainfall, while the influence of the spatial resolution of the rain field and of the rainfall network density is studied in sections Sensitivity to the spatial resolution of the rain fields and Influence of the network density, respectively. In the concluding section Discussion and conclusion, the results of these various sensitivity analyses are summarized, before discussing their validity and limits.

**Studied area, data and hydrological model**

**The CATCH-Niger observing system**

A major obstacle when dealing with the scale issue is the lack of data sets covering a sufficiently large range of scales. This is especially true in tropical Africa where hydro-meteorological networks have suffered a constant degradation since the mid 1980s. To our knowledge, the CATCH-Niger observing system is the only data set in the region where high resolution (in time and space) rainfall data cover an area of several thousand km², with concomitant runoff measurements on small catchments. As described in B06, the CATCH-Niger observatory is located between 13°N and 14°N and covers 110 x 160 km² around Niamey (Niger). On this area, a permanent network of thirty recording rain gauges has been available since 1990. Over the studied period 1990-2002, 548 mesoscale rainfall events were recorded at a time step of 5 min. These events, associated with large organized convective systems (OCSs) moving from East to West, were shown to account for more than 85% of the annual rainfall over the region (see B06). OCSs are formed by a convective front displaying great spatial variability and a trailing stratiform area of low and relatively homogeneous rainfall. The hyetograph associated with the passage of such a system over a given location has a very typical shape, made of one to sometimes two rainfall peaks lasting between 30 and 60 min, followed by a lighter rain lasting for a few hours (see Fig. 10 in B06).

Regarding surface hydrology, the main hydrological feature of the CATCH-Niger area is endoreism. From the survey of several endoreic catchments and pool outlets, Desconnets et al. (1997) and Peugeot et al. (2003) estimated that less than 2% of the rain falling over the region covering the left bank of the Niger river flows to the river, which means that this area may be considered to be confined as far as computing a water budget is concerned. Peugeot et al. (1997) showed that, as in most semi-arid regions, runoff is intermittent and exclusively of Hortonian origin. Because the depth to water table is several tens of meters below the surface (Leduc et al., 1997) and there is no base flow feeding the pools, they are exclusively filled up by direct runoff. Since the pools constitute a major contributor to the recharge of the regional aquifer (Desconnets et al., 1997; Leduc et al., 1997), runoff at catchment scale is a central component for assessing the water balance over the region.

**Hydrological model used**

In the Sahel, Peugeot et al. (1997) computed the parameters of a simple SCS-like threshold method (Soil Conservation Service, 1972) from field measurements taken from 1991 to 1994 on several catchments. By comparing measured and simulated runoff they obtained regression coefficients greater than 95% showing that the runoff of Sahelian catchments is well represented by such a model. On the Wankama endoreic catchment (1.8 km² inside the CATCH-Niger area), a comparison of this simple model with a more sophisticated model using an explicit resolution of the
Green & Ampt equation for infiltration (this model is derived from the hydrological model r.water.fea, see e.g. Vieux, 2005), was carried out over the 1990–2003 period. This comparison showed that both models were producing similar runoff (Massuel, 2005). This is in line with the similar study of Michaud and Sorooshian (1994b) on the Walnut Gulch catchment (150 km², Arizona). It will be consequently considered in this paper that the simple threshold model provides an appropriate representation of the small endoreic watersheds of the region, for the purpose of studying the sensitivity of runoff production. 

The main modification brought by Peugeot et al. (1997) to the classical SCS model is the computation of an effective rainfall variable, based on the concept that runoff will occur only when rain intensities are above a threshold — 5-min rainfall. This is linked to the ability of the catchment to store water, especially for the event of 18 July 1997 recorded at the Niamey station. All 5-min rainfall below the threshold \( I_t \) are eliminated, and \( P_u \) is the sum of the remaining 5-min rainfall.

\[
R = (P_u - P_i)^2 / (P_u - P_i + S) \quad \text{if} \quad P_u > P_i \\
R = 0 \quad \text{if} \quad P_u < P_i
\]

with

\[
P_u = \sum_{i=1}^{L} (I(t_i)/I(t_i) \geq I_t)
\]

In Eqs. (1) and (2) \( R \) is the runoff depth over the catchment, \( S \) is linked to the ability of the catchment to store water, and \( P_i \) represents the initial infiltration before runoff starts. \( P_u \) is the total effective rainfall for the event calculated in Eq. (3) (Fig. 1), where \( I_t \) is the rainfall intensity threshold over which the rain water runs off, \( L \) is the number of 5-min time steps \( t_i \), and \( I(t_i) \) is the rainfall intensity at time step \( t_i \) for the event under consideration.

Obviously there is a dependence between the parameters \( I_t \) and \( P_i \) that both represent the initial losses in the catchment. However \( I_t \) is interesting because it explicitly represents the role of the time variability of the rainfall intensities in the production of runoff, as was observed by Peugeot et al. (1997). Peugeot et al. (1997) already mentioned the dependence between \( P_i \) and \( I_t \) and have fixed \( P_i \) to zero for their test catchments meaning that \( P_i \) could be sometimes insignificant compared to \( I_t \) in the formulation of the model. However in the present study it was decided to keep \( P_i \) as a third parameter, because \( P_i \) was shown to be useful to calibrate the rainfall–runoff relationship from the measured data available on the few available gauged catchments. In the same region, the three parameters formulation was also kept by Massuel (2005) for the same reason: keeping both \( P_i \) and \( I_t \) as parameters allows improving the performance of the hydrological model.

Three sets of parameter values characterize as many types of catchments, considered as being representative of the catchments of the area (Peugeot et al., 1997; Massuel, 2005):

- Set #1 (\( P_i = 0 \) mm, \( S = 39 \) mm, \( I_t = 12 \) mm/h) — watershed type 1, high runoff coefficient;
- Set #2 (\( P_i = 10 \) mm, \( S = 64 \) mm, \( I_t = 12 \) mm/h) — watershed type 2, moderate runoff coefficient;
- Set #3 (\( P_i = 20 \) mm, \( S = 70 \) mm, \( I_t = 12 \) mm/h) — watershed type 3, low runoff coefficient.

The rainfall–runoff scatter plots obtained when running these 3 models on the 548 rain events are shown in Fig. 2. Because of the independent functioning of the small endoreic catchments, the water balance may be considered, in a first approximation, as the addition of the water budgets computed for each endoreic entity. Since it is impossible to carry out observations on all the hundreds of small catchments covering the region, the idealized modeling strategy proposed hereafter consists of creating an idealized hydrological environment made of the adjunction of virtual 1 \( \times \) 1 km² catchments. The rainfall–runoff model is then run on each catchment, assuming that all the catchments are either of type 1, type 2, or type 3. The exact protocol differs depending on whether one considers the sensitivity to the temporal or the spatial variability of rainfall.

**Methodology: an idealized approach of the scale issue**

Three components are combined in this study: (i) a set of rain fields computed for the 548 rainfall events at various time and space resolution; (ii) the SCS-type rainfall–runoff model used for simulating the effect of the rainfall variability on runoff generation; (iii) a so-called “idealized strategy”, building on the fact that in this part of the Sahel, the hydrological environment is a mosaic of small endoreic units behaving as independent water budget units.

**Protocol description**

Effect of temporal variability inside rainfall events

The hydrological model is an event-based model; however, it takes into account the time variability at smaller scales by computing effective rainfall which is the potential rain depth available for runoff (see Eq. (3) and Fig. 1). This involves a threshold effect that is dependent on the time variability of intensities inside rain events. The influence of the time rainfall variability is evaluated by testing two synthetic hyetograph models based on rectangular and triangular shape in comparison with the observed hyetograph. One is the trivial rectangular – or mean intensity – hyetograph (referred to as MIH hereafter). The other hyetograph model was initially proposed by Guillot and Lebel (1999) and reformulated in B06. It is made by a triangular shape that follows a rectangular shape and was shown to well represent, respectively, the convective and stratiform parts of the average hyetograph associated with the organized convective systems (referred to as OCSH hereafter) (see Fig. 11 in B06).

The protocol used to evaluate the runoff sensitivity to time rainfall variability is as follows:

1. Idealized hydrological environment (Fig. 3a). Co-located with each of the 30 rain gauges of the CATCH-Niger network, we assume the presence of a “virtual” 1-km² endoreic catchment with a rainfall–runoff relationship defined by Eqs. (1)–(3).

![Fig. 3](https://example.com/fig3.png)

*Fig. 3* Design of the idealized sensitivity experiment. (a) Idealized environment for temporal sensitivity analysis, composed of 30 “virtual” 1-km² catchments centered on the rain gauge locations. (b) Idealized environment for spatial sensitivity analysis, composed of 10,000 “virtual” 1-km² catchments located on a regular grid over a 100 × 100 km² area.

**Please cite this article as:** Théo Vischel, Thierry Lebel, Assessing the water balance in the Sahel: Impact of small scale rainfall variability on runoff, Journal of Hydrology (2006), doi:10.1016/j.jhydrol.2006.09.007
Simulation of a reference runoff. For a given rain event, 5-min rain series recorded at each of the 30 rain gauges are used to run the idealized rainfall–runoff model with constant parameters $P_i$, $S$, and $l_i$. A "reference" value of the event runoff is then computed for this rain event by averaging the 30 runoff depth values obtained on each of the 30 "virtual" catchments.

(iii) Sensitivity analysis.

For the same event, the two tested synthetic hyetographs MIH and OCSH are implemented. Note that these two types of hyetographs preserve the event rain depth measured at each one of the 30 rain gauges and only differ on the distribution of the rainfall intensities over the event duration.

The rainfall–runoff model is forced with the two different event runoff values by averaging the simulated runoff depths over the 30 virtual catchments.

(iv) Repetition for all events. Points ii and iii are repeated independently for all of the 548 rain events recorded over the 1990–2002 period. The 548 reference runoff depth values obtained from the two series of 548 runoff depth values obtained from the two synthetic hyetographs.

Effect of decreasing the spatial resolution of event rain fields

The protocol used to evaluate the runoff sensitivity to spatial resolution of rain fields is as follows:

(i) Idealized hydrological environment (Fig. 3b). A 10,000-km$^2$ area is defined inside the CATCH-Niger study area that is assumed to be made of 10,000 square grid cells of 1 km$^2$ defining "virtual" endoreic catchments with a rainfall–runoff relationship defined by Eqs. (1)–(3).

(ii) Simulation of a reference runoff.

For a given rain event, a reference high resolution (1 $\times$ 1 km$^2$) mesoscale rain field is computed by kriging-interpolation of the 30 rain gauge measurements. The variogram used is presented in Figure 3b. This problem will be treated here in two steps.

First, the effect on runoff of a decrease in the number of rain gauges homogeneously scattered over the entire area is not preserved. The runoff at coarser resolution, when decreasing the network density, the event runoff obtained from the rain fields at coarser resolution. An event mesoscale runoff value is obtained by averaging the 10,000 runoff depth values obtained on each of the 10,000 "virtual" catchments.

(iv) Repetition for all events. The simulation is carried out in an independent way for each of the 548 rain events recorded over the 1990–2002 period. The 548 reference runoff depth values obtained from the 1 $\times$ 1 km$^2$ resolution rain fields are compared to the series of 548 runoff depth values obtained from the rain fields at coarser resolution.

Seven resolutions are tested in this protocol: the 1 $\times$ 1 km$^2$ reference resolution and 6 coarser resolutions: 5 $\times$ 5 km$^2$, 10 $\times$ 10 km$^2$, 20 $\times$ 20 km$^2$, 25 $\times$ 25 km$^2$, 50 $\times$ 50 km$^2$, and 100 $\times$ 100 km$^2$.

Note that the time disaggregation from the event time step to the 5-min time step is realized by applying a synthetic hyetograph on each pixel (i.e. each "virtual" catchment). In a first approximation (section Results) the hyetograph will be applied as an point hyetograph not accounting for the propagation of rainfall systems. Then in section Effect of the time disaggregation in conjunction with special aggregation, the propagation of the rainfall systems will be taken into account to determine how it influences the results.

Effect of spatial sampling of event rain fields

In the protocol presented above related to the spatial resolution of the forcing rain field, it is implicitly assumed that the reference high resolution rain field is perfectly known. This is obviously not the case, and one may thus question the extent to which the results will depend on the uncertainties associated with the reference high resolution rain field. This problem will be treated here in two steps.

First, the effect on runoff of a decrease in the number of stations used for computing the areal rainfall and for interpolation is considered. In line with the sensitivity study carried out in B06, four sub-networks composed of 21, 12, 8, and 4 rain gauges homogeneously scattered over the 10,000-km$^2$ area are built. High resolution (1 $\times$ 1 km$^2$) rain fields are built from these networks and used to force the hydrological model following the points i and ii of the protocol described above. Contrary to a decrease of the spatial resolution, when decreasing the network density, the event areal rainfall over the entire area is not preserved. The runoff values simulated from these four sub-networks are then compared to the reference runoff obtained from the rain fields elaborated with the initial 30-rain-gauge network.

Then the protocol related to the spatial resolution is entirely followed by replacing the reference rain fields elaborated at 1 $\times$ 1 km$^2$ resolution with the 30-rain-gauge network by reference rain fields elaborated at 1 $\times$ 1 km$^2$ resolution with 49 rain gauge networks (with $N = 21$, 12, 8, and 4, corresponding to the four sub-networks). The effects of spatial
Discussion on the approach used

Obviously one may discuss the assumptions behind this idealized modelling exercise. Some of these assumptions — mostly those related to rainfall, namely the points related to spatial kriging-interpolation, aggregation and time disaggregation — have a strong foundation in previous work (e.g. Lebel and Amani, 1999; Guillot and Lebel, 1999). The other assumptions, related to runoff production, are more tentative. They had to be made because only two catchments have been monitored continuously during the past ten years, which means that it was not possible to evaluate precisely the surface of each of the small endorheic catchments. It was also impossible to calibrate the threshold model for each of these catchments, thus necessitating the idea of an idealized modelling exercise with the assumption that all the watersheds are the same size and have the same surface conditions. We have to consider the possible implications of these assumptions on the results of our study:

Assumption of equal size of the catchments. In their study, Peugeot et al. (1997) showed that the larger the catchment the smaller the runoff per surface area due to increased channel losses, a fact already noted by Goodrich et al. (1997). Our study will show that the smaller the runoff coefficient, the larger the sensitivity to the resolution of the input rain field. Thus, using a reasonably small size for the catchments guarantees that the sensitivity will not be overestimated.

Surface conditions are different from one catchment to another, which means that the parameters of the rainfall—runoff model should vary. To evaluate the influence of this variability on the results of our study, the three sets of parameters listed above in section Hydrological model were chosen — deemed to cover a representative range of possible values for the area — and the whole sensitivity experiment was run for these different sets of values.

Independent simulation of the runoff events. Peugeot et al. (1997) and Galle et al. (1999) have shown that the initial soil humidity is a second order factor in explaining the surface runoff in the Niamey area. This is because the initial soil humidity decreases rapidly after the end of the rain event, and the interval between rain events is greater than one day, even during the peak of the rainy season.

Numerical criteria

Given the general framework of the study — evaluating the sensitivity of water balance assessment to the resolution of the rain fields used in rainfall—runoff modelling — and the simplification linked to its idealized nature, the analysis will focus on the total runoff volume at the event scale over the whole study area (average of the 30 stations for the temporal analysis, average of the 10,000 pixels for the spatial analysis), rather than on finer resolution variables such as peak runoff or time to peak for individual catchments.

A reference value of the total runoff for the study area (see point ii of the two protocols in 3.1) and event k is computed with the 5-min time step hyetograph for the temporal analysis and with the 1 × 1 km² rain field for the spatial analysis:

\[ R_k = \sum_{j=1}^{N_{ec}} r_{jk} \quad (k = 1, 548) \]

with \( N_{ec} \) the number of endorheic catchments (\( N_{ec} = 30 \) in the temporal sensitivity analysis and \( N_{ec} = 10,000 \) in the spatial sensitivity analysis), and \( r_{jk} \) being computed for each virtual catchment using the rainfall—runoff model described in section Hydrological model used. Then the synthetic hyetographs and decreased resolution rain fields are used to compute alternate values \( r_{jk} \) and their sum \( R'_k \):

\[ R'_k = \sum_{j=1}^{N_{ec}} r'_{jk} \quad (k = 1, 548) \]

The main objective of the analysis is to compare the reference series \( \{R_k\} \) to the various simulated series \( \{R'_k\} \). At the event time step, four statistical indicators are used (their formulas are given in Appendix), namely the coefficient of determination of the linear correlation between the two series \( r^2 \), the Nash criterion \( e_N \), and the normalized RMSE (RSME). The global error on mean annual runoff is simply quantified by the Relative Error (RE). In addition to computing these 4 criteria, attention will be paid to the respective distributions of \( \{R_k\} \) and \( \{R'_k\} \).

Sensitivity to time variability and hyetograph modelling

Using the protocol described in section Protocol description related to the effect of time variability, the runoff values produced by the two models of hyetographs MIH and OCSH are compared for each of the 548 rain events to the runoff produced by the observed hyetograph.

Table 1 shows the value of the \( r^2 \), \( e_N \), and RE criteria for the three sets of parameters. Using the OCSH instead of the observed hyetograph leads to a 9–15% overestimation of the mean annual runoff. This overestimation is mainly due to the highest events for which the single peak model is not well suited. Improvements of this model have been recently proposed by Messager et al. (in press). However the performance of the OCSH are largely better than that of the MIH. Despite the good correlations computed for the two models of hyetograph, there is a serious bias in the runoff generated through the MIH. The efficiency \( e_N \) is inferior to the correlation \( r^2 \), and the relative error ranges from 19% to 65% (versus a 9–15% range for the OCSH). The bias introduced by the MIH is highly dependant on the set of parameters used, while the bias from the OCSH is less sensitive to this factor.

Our results are in good agreement with the works of Lambourne and Stephenson (1987), already cited above, who found that the rectangular hyetograph overestimated the runoff, while runoff was better simulated when respecting the hyetograph shape.
Such runoff sensitivity to the hyetograph shape shows how representing the main variabilities of the hyetograph intensities is important in assessing the runoff volumes. For this reason, the OCSH was used to disaggregate in time the event rain fields as discussed in the next section.

These results are directly linked to the structure of the hydrological model and its parameterization. The value of the intensity threshold $I_t$ is particularly determinant. The lower the threshold value, the lower the sensitivity of the model output to the representation of the time structure. The 12 mm/h value is used in this study as a mean representative value of the threshold that could in fact vary between 7 and 18 mm/h (Peugeot et al., 1997). Varying the value of $I_t$ in the interval (7 – 18 mm/h) did not bring any significant change to our conclusions. It is worth noting, however, that the higher the runoff coefficients (i.e. parameter set #3), the greater the sensitivity of simulated runoff to time variability of rainfall.

<table>
<thead>
<tr>
<th>Hyetograph model</th>
<th>$r^2$</th>
<th>$e_n$</th>
<th>RE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set #1 Kr = 25.1%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MIH</td>
<td>0.99</td>
<td>0.90</td>
<td>19</td>
</tr>
<tr>
<td>OCSH</td>
<td>0.96</td>
<td>0.91</td>
<td>9</td>
</tr>
<tr>
<td>Set #2 Kr = 9.3%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MIH</td>
<td>0.98</td>
<td>0.73</td>
<td>45</td>
</tr>
<tr>
<td>OCSH</td>
<td>0.95</td>
<td>0.90</td>
<td>11</td>
</tr>
<tr>
<td>Set #3 Kr = 4.1%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MIH</td>
<td>0.95</td>
<td>0.60</td>
<td>65</td>
</tr>
<tr>
<td>OCSH</td>
<td>0.94</td>
<td>0.90</td>
<td>15</td>
</tr>
</tbody>
</table>

Sensitivity to the spatial resolution of the rain fields

The results of the protocol used to study the effect of rain field spatial resolution — as described in section Protocol description — is illustrated by an example in Fig. 4 for one

Fig. 4 Impact on runoff of the spatial aggregation of the forcing rain field: results obtained on the rainfall event of 12 August 1994. The parameter set used in this example is #2 (intermediate average runoff coefficient $Kr = 9.3\%$).
set of parameters of the SCS-type model (set #2). The initial rainfall field and the 6 aggregated rainfall fields are shown for the rain event of 12 August 1994 (total rain depth of 16 mm over the study area), along with the derived ‘runoff off fields’. It is important to note that although the aggregation process maintains the mean spatial rainfall, the smoothing of the rainfall intensities and the non-linearity of the rainfall–runoff relationship result in a gradual decrease of the mean spatial runoff as the resolution increases.

Results

The statistics given in Table 2 quantify the effects of the procedure applied to the 548 rain events. The corresponding mean annual runoffs are plotted in Fig. 5, both in absolute values and relative to the runoff computed from the original full resolution rain fields. In general (92% of the 548 cases for parameter set #1, 95% for parameter set #2, 100% for parameter set #3), the runoff computed over the whole study area decreases when the rain field resolution is coarser.

As seen from Table 2, the determination coefficient \( r^2 \) is always greater than 0.9. Thus, according to \( r^2 \), the impact of decreasing the rain field resolution is not significant. On the other hand, the Nash coefficient displays much more sensitivity, resulting from the bias generated when decreasing the resolution. This bias is always negative (a logical consequence of having a vast majority of events for which the runoff decreases when the resolution decreases) and reaches 50% in the worst case (coarser resolution and parameter set associated with the smallest runoff coefficients). At the reference resolution of 1x1 km², the mean annual runoff is 110 mm for parameter set #1, 32 mm for parameter set #2, and 11 mm for parameter set #3. These values decrease to 93 mm, 21 mm, and 4 mm, respectively, at the coarsest resolution of 100x100 km² (Fig. 5a).

Table 2 also shows that the number of non-zero runoff events is highly sensitive to aggregation. For parameter set #1, 543 events have a non-zero runoff over the study area when using the full resolution 1x1 km² forcing rain fields; this value decreases to 426 when working at the 1x1 km² resolution, which means that 22% of the events showing some runoff at the full resolution have zero runoff over the study area when assuming a spatially uniform rainfall. For parameter sets #2 and #3, the values are, respectively, 447 and 162 (64% of the non-zero runoff events become zero-runoff events) and 319 and 52 (84% of the non-zero runoff events become zero-runoff events).

The less intense a rainfall event, the sooner the runoff

![Fig. 5](image)

Fig. 5  Effect of the spatial aggregation on the mean annual runoff. (a) Evolution of simulated runoff as a function of resolution. (b) Evolution of the percentage of the total runoff as a function of resolution. The reference total runoff is the mean annual runoff simulated based on the reference 1x1 km² resolution rain fields.

becomes zero in the progressive aggregation procedure. The main effect of using coarse resolution forcing rain fields is thus a significant underestimation of runoff.

The graphs of Fig. 6 illustrate the change in the runoff distribution resulting from degradation of the resolution of the forcing rain fields. For set #1 at the $1\times1$ km$^2$ resolution, 90% of the runoff is explained by rainfall events greater than 10 mm, representing 42% of the total number of events and 77% of the total rainfall. At the coarsest resolution, about 30% of the rainfall events (representing 65% of the total rainfall) contribute to 90% of the runoff. For set #3 at the $1\times1$ km$^2$ resolution, 90% of the runoff is explained by rainfall events greater than 20 mm, representing 18% of the 548 events and 47% of the total rainfall. At the coarsest resolution, about 5% of the rainfall events (representing 17% of the total rainfall) contribute to 90% of the runoff (and 100% of the runoff is explained by the rainfall events greater than 25 mm, representing only 10% of the total number of events). This means that the intense events are generally less sensitive to the resolution effect, because they are relatively less spatially variable than the small events. This is illustrated in Fig. 7, which shows that the spatial coefficient of variation (the spatial standard deviation of the rain field divided by the event rainfall) is a decreasing function of the resolution.

Fig. 6 Cumulative runoff distribution expressed as a function of the event rainfall for various resolutions of the forcing rain fields and the three sets of parameters of the hydrological model. The dashed line on graph (a) shows how to read the graph: for the $1$-km$^2$ resolution, the rainfall events greater than 24 mm represent only 11% of the 548 events, but they explain 50% of the total runoff. The associated right hand graph shows that these 11% of events produce 33% of the total rainfall.

event rain depth. This is concordant with the work of Winc- 703 hell et al. (1998), showing that the decrease in runoff due to 704 rain field spatial aggregation depends on the spatial varia- 705 tion of the maximum hyetograph intensities. In our Sahelian 706 case, a strong correlation exists between the peak 5-min 707 rain intensity recorded at a given location and the total 708 event rain depth at the same location (Guillot and Lebel, 709 1999).

The various statistics presented above show that the sen- 710 sitivity to the rain field resolution is stronger for lower run- 712 off coefficient catchments. Such catchments are indeed 713 characterized by a strong non-linearity in the rainfall–run- 714 off relationship so that a change in rainfall input is accentu- 715 ated in runoff output as mentioned by Shah et al. (1996). As 716 in section Sensitivity to time variability and hyetograph mod- 717 elling, one may argue that the results are largely depen- 718 dent on the value of the threshold parameter \( I_t \). However, 719 the two values, 7 mm/h and 18 mm/h, identified by Peugeot 720 et al. (1997) as limit values were also tested in the model 721 (instead of the mean 12 mm/h value) without changing sig- 722 nificantly the overall pattern of our results.

Although the effect of decreasing resolution depends on 723 the various factors analyzed above, the relative errors given 724 in Table 2 and the plot of Fig. 5b suggest that the critical 725 resolution for which errors become significant is somewhere 726 between 100 km² and 625 km², corresponding to a charac- 727 teristic length in the interval [10–25] km. By comparison, a 728 characteristic length of 20 km was found by Booij (2003) 729 considering extreme precipitation assessment in Western 730 Europe. As far as runoff estimation is concerned, a 10-km 731 resolution for the forcing rain field appears in our case to 732 be on the safe side, while a 20-km resolution appears to still 733 be acceptable (bias lower than 10% for all three sets of 734 parameters).

The determination of such characteristic scales is of 735 course largely dependant on both the intrinsic covariance 736 structure of rainfall and the density of the rain gauge net- 737 work available to compute the rain field. This point will 738 be further addressed in section Influence of the network 739 density.

Effect of the time disaggregation in conjunction 742 with spatial aggregation

The temporal pattern of the point hyetograph and that of 744 the corresponding areal hyetograph are not identical. In 745 section Results, a model of a point hyetograph – described

![Figure 7](image1.png)

**Fig. 7** Spatial coefficient of variation of the 548 event rain fields, as a function of the event rainfall.

![Figure 8](image2.png)

**Fig. 8** Example (event of 4 August 1999) of the areal hyetographs computed over cells of increasing surface area by taking into account the westward propagation of the OCSs.
Influence of the network density

Effect of rainfall spatial sampling error on runoff

Contrary to the decrease of the resolution that conserves the event areal rainfall, the decrease of the network density will not generally preserve the event areal rainfall (it is preserved in the mean but not necessarily for a given event). Taking the rainfall and runoff computed from the 30-rain-gauge network as a reference, the root mean square errors for the four sub-networks defined in B06 are given in Table 3. The rainfall sampling errors are of the same order as the mean errors computed from the more systematic sampling error study in B06 (see Table 3 in B06) (in the present study, no separation was made between the pre-onset events and the core of the rainy season events). Comparing rainfall and runoff, it is seen that the errors on rainfall and runoff remain comparable for the 21-gauge, 12-gauge, and 8-gauge networks. However, for the less dense network of 4 stations, the rainfall RMSE of 48% is amplified into a runoff RMSE of 155%. Remember that the errors due to under-sampling may result from two opposite effects: (i) either the network misses the small rain cells and thus underestimates the rainfall and associated runoff, or (ii) some stations of the network catch small cells and, due to the spatial interpolation, isolated high intensities are spread over a large area, leading to overestimation of rainfall and runoff. Occasional, either of these two effects may generate considerable errors in the rain field, and these errors are more frequent with the low density network. These large errors are then amplified by the non-linearity of the rainfall–runoff model. The dramatic jump of the runoff RMSE observed when decreasing the number of stations from 8 to 4 is a direct consequence of a strong increased probability of occurrence of either of these two effects when working with only 4 stations over an area of this size. This corresponds to the upper limit of the density of operational rain gauge networks in this region.

Link between the density of network and aggregation effect

The question addressed now is how the density of the original rain gauge network influences the resolution effects studied in section Results. To answer this question, the aggregation procedure used in section Results based on the information provided by the 30-rain-gauge network is similarly used for the sub-networks created in section Effect of rainfall spatial sampling error on runoff above. While in
Denoting $D_{\text{net}}$ as the smallest of the above characteristic scales, $D_{\text{net}}$ as the resolution of the rain gauge network (typically the size of the grid mesh of a regular network), and $D_I$ as the resolution of the rain field used to force the hydrological model, two main situations may arise that are also discussed in other studies (see e.g. Bloschl and Sivapalan, 1995; Western and Bloschl, 1999):

- In time, the main requirement is to correctly separate the convective and stratiform components of the hydrograph. Failing to do so may lead to a 65% overestimation of runoff over the entire study area in the worst case.
- When considering a mosaic of small catchments covering several thousand km$^2$, the global hydrological response is highly sensitive to the spatial resolution of the rain fields used to force the model. The global runoff averaged over the 548 events may be underestimated by 50% in the worst cases.
- A resolution of 20 km, corresponding to a characteristic spatial scale, is found to constitute a threshold; for coarser resolutions, the performance of the model rapidly decreases.
- The effect of spatial aggregation depends on the density of the rain gauge network, with lower effect for sparser networks.
- The catchments characterized by low runoff coefficients are more sensitive to the time and spatial variability.
911 Validity of the results
912
913 Three points have to be considered to assess how meaningful are the results presented here: numerical experiment strategy, model parameters, statistical significance.
914
915 Numerical experiment strategy. As explained in section 916 Discussion of the approach used, the approach used to simulate the runoff at the mesoscale results from the very specific endoreic hydrological environment of the region of interest. This type of environment is characterized by a multitude of small size independent catchments. Since it is not possible to monitor the hundreds of catchments of the study area, one has to use the knowledge acquired on a limited number of instrumented catchments to construct a plausible, though strongly idealized, representation of that environment. In appearance, the strongest idealization is to assimilate each catchment to a square of constant area, whereas in reality the shape and area are much more diverse. There are good reasons to believe, however, that this simplification is not unrealistic, especially because in this particular context, surface runoff may be considered to be an additive process. Thus, the exact shape and size of the catchments are probably not so important as long as one is interested in the annual water balance rather than in a precise restitution at smaller time scales.
918
919 Model parameters. A probably more disputable point relates to the parameters of the model. For sake of simplicity — and thus easier analysis of the results — we worked with three sets of parameters. Each set was uniformly applied to all the ideal catchments. The reality is a complex mix of many different sets, and it is difficult to represent it without an appropriate knowledge of the surface conditions that determine the parameter values of the SCS model. Work is underway to acquire this knowledge on a 1760-km² sub-area from remote sensing and Shuttle Radar Topography Mission (SRTM) data (Farr and Kobrick, 2000). The first results of this work, which allows working in more realistic conditions but on a much smaller area, show that the sensitivity of runoff to spatial resolution was stronger than the sensitivity displayed when using set #2 (Kr = 9.3%) and lower than the sensitivity displayed when using set #3 (Kr = 4.1%). This confirms that the simple approach adopted in the present paper gives representative orders of magnitude of the simulated runoff sensitivity to spatial resolution effects. It also confirms that the SCS-type models are rightly assumed to be a good representation of the semi-arid Hortonian catchments, and that, using a few sets of parameters spanning a large range of runoff coefficients, one can obtain a tractable macroscopic vision of the importance of accounting for rain field resolution effects in rainfall–runoff studies.
928
929 Statistical significance. Based on a systematic use of a finite resolution data base of 548 rain events spanning a wide range of climatic conditions (dry years, wet years, pre-monsoon events, and monsoon events), the results presented here are highly significant from a statistical point of view.
932
933 Recommendations
934
935 The results obtained in this study show the importance of taking into account the convective scale variability of rainfall in semi-arid regions to evaluate the impact of climate variability and climate change on water resources. In many parts of the Sahel, water is a local resource in an endoreic environment. There is no active large scale river system acting to smooth out the rainfall variability, which thus directly impacts the underground water resources. Consequently, a natural follow-up to this study is to investigate whether rainfall disaggregation models allow a sufficiently realistic restitution of the rain field variability to be used as intermediate models between large scale climate models and small scale rainfall–runoff models.
940
941 Acknowledgements
942
943 This research was funded by IRD in the framework of the AMMA-CATCH “ORE” program initiated by the French Ministry of Research, now supported by the Institut des Sciences de l’Univers (INSU). Special thanks are due to Dr. Baxter Vieux who provided his expertise in hydrologic modeling at the initial stage of this project and who kindly accepted with help of Ami Arthur to review this paper for English syntax. We would also like to acknowledge the contribution of our colleagues at HSM (Hydro-Sciences Montpellier), B. Cappeleare, C. Peugeot and L. Seguis for a long standing collaboration on the AMMA-CATCH project.
951
952 Appendix. Variables:
953
954 \( N_{\text{evt}} \) the number of rainfall events (\( N_{\text{evt}} = 548 \)).
955 \( \bar{R} \) are reference runoff values. \( \bar{R} \) is the average of the \( N_{\text{evt}} \) reference runoff values.
956 \( R_i \) are the runoff values to be compared with the reference runoff values. \( \bar{R} \) is the average of the \( N_{\text{evt}} \) runoff values to be compared.
957
958 Numerical criteria:
959
960 – Determination coefficient:
961 \( r^2 = \frac{\sum_{k=1}^{N_{\text{evt}}} (R_k - \bar{R}) (\bar{R}_k - \bar{R})^2}{\sum_{k=1}^{N_{\text{evt}}} (R_k - \bar{R})^2 \sum_{k=1}^{N_{\text{evt}}} (\bar{R}_k - \bar{R})^2} \)
962
963 – Nash criterion:
964 \( e_N = 1 - \frac{\sum_{k=1}^{N_{\text{evt}}} (R_k - \bar{R}_k)^2}{\sum_{k=1}^{N_{\text{evt}}} (R_k - \bar{R})^2} \)
965
966 – Normalized RMSE:
967 \( \text{RMSE} = \sqrt{\frac{1}{N_{\text{evt}}} \sum_{k=1}^{N_{\text{evt}}} (R_k - \bar{R}_k)^2} \)
968
969 – Relative error RE:
970
References


Assessing the water balance in the Sahel: Impact of small scale rainfall variability on runoff 15


