

Le modèle hillslope-storage Boussinesq

:

Approche et perspectives

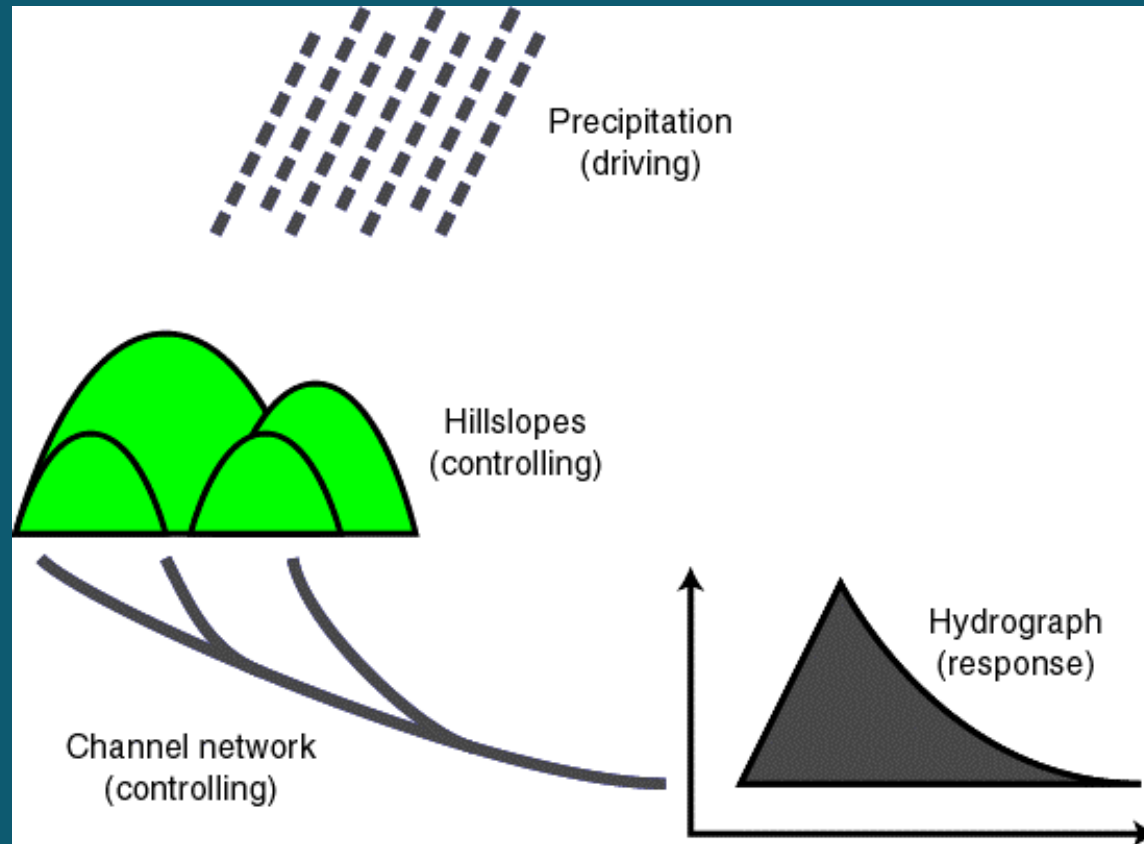
A. Berne, R. Uijlenhoet and P. Troch

Wageningen University

The Netherlands

Context

Catchment hydrological system



Research questions

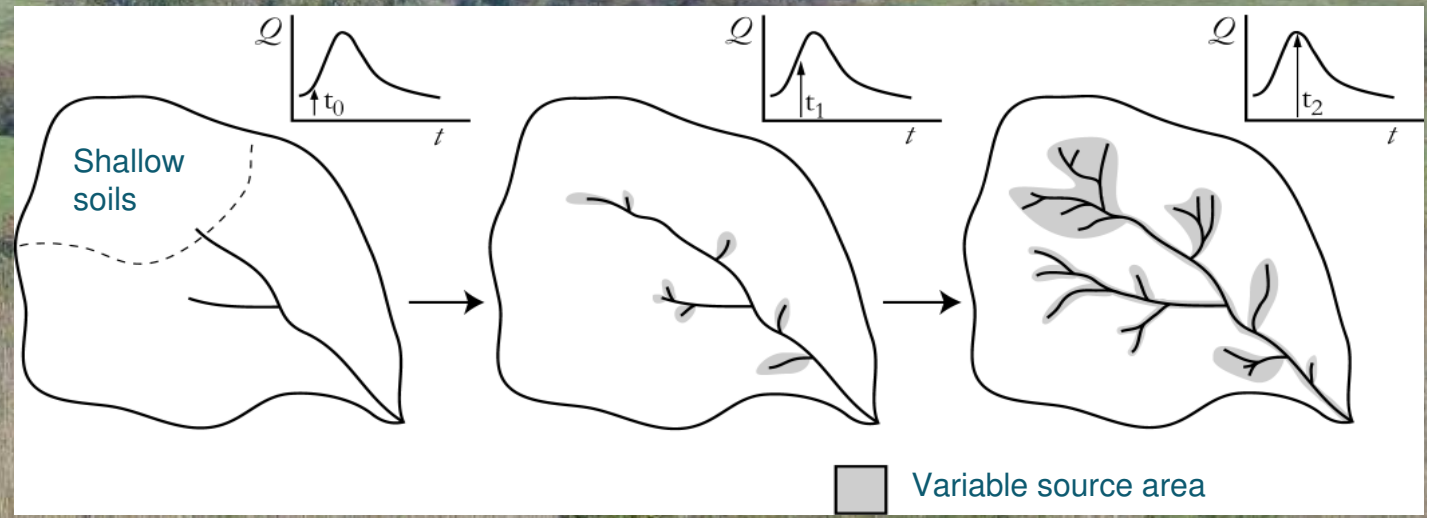
- Is it possible to describe the behaviour of the system using simple models of the hydrological processes at the hillslope scale?
- Can we link the characteristics of the subsurface flow to the geomorphological and pedological properties of the hillslope?

The hillslope-storage Boussinesq approach

Humid regions with highly conductive thin soils.

Runoff mainly comes from saturation excess mechanisms.

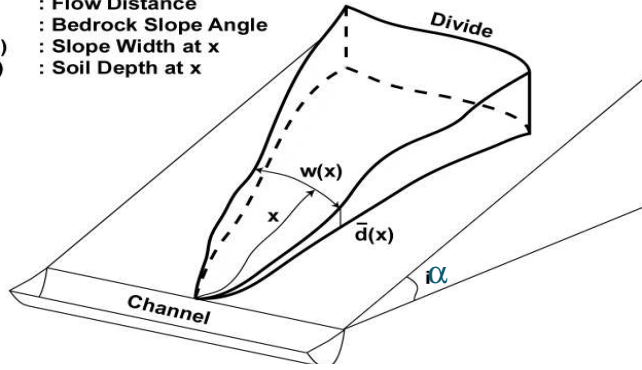
Subsurface flow processes must be carefully modelled.



HsB equation

Convergent Hillslope

- x : Flow Distance
- i : Bedrock Slope Angle
- w(x) : Slope Width at x
- d(x) : Soil Depth at x

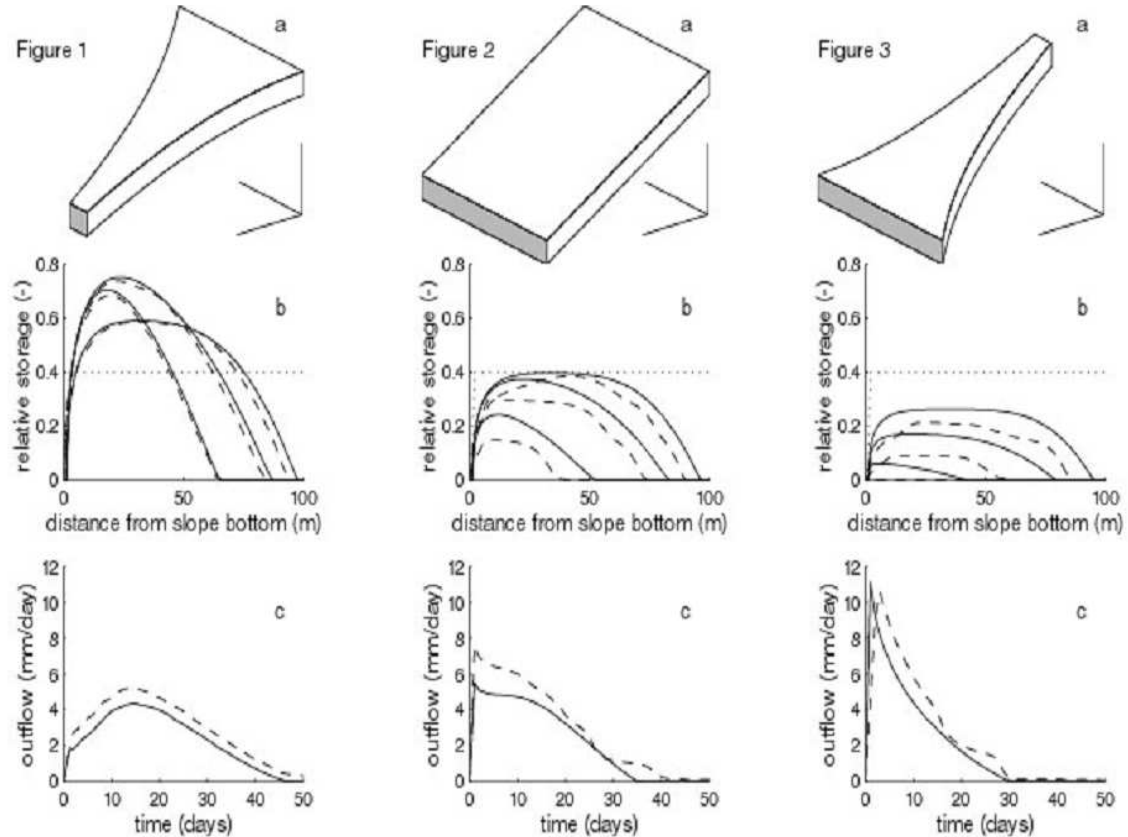
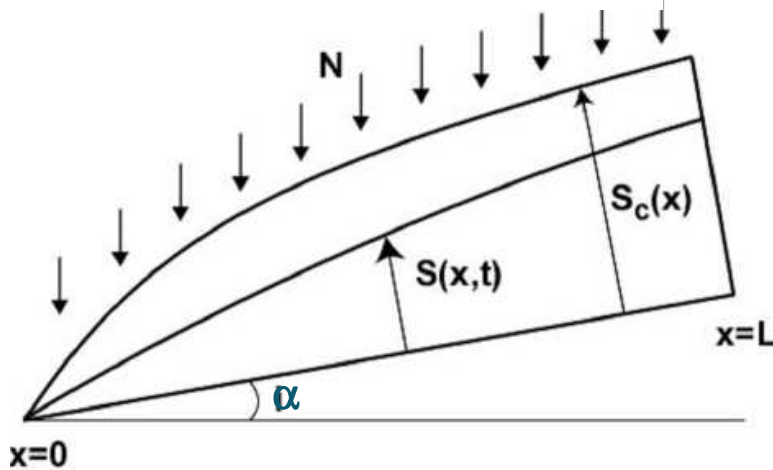


$$f \frac{\partial S}{\partial t} = \frac{k \cos \alpha}{f} \frac{\partial}{\partial x} \left[\frac{S}{w} \left(\frac{\partial S}{\partial x} - \frac{S}{w} \frac{\partial w}{\partial x} \right) \right] + k \sin \alpha \frac{\partial S}{\partial x} + fN(t)w$$

Comparison with 3D Richards' equations

$$S_c(x) = w(x) \bar{d}(x) f$$

$$S(x,t) = w(x) \bar{h}(x,t) f$$

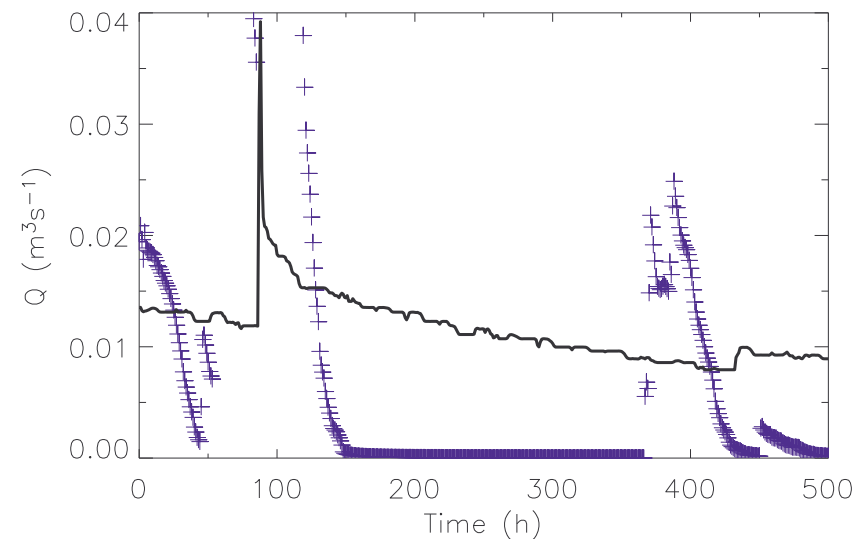
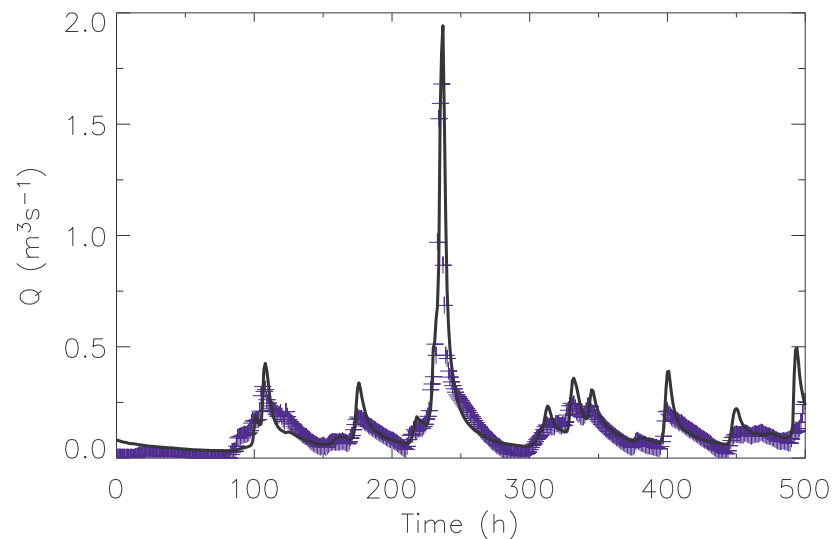


1D Hillslope-storage Boussinesq (hsB) equation

HsB on a real catchment

Tanllwyth catchment (0.97 km², Plynlimon, Wales)

- 5 years of hourly rain and stream measurements.
- DEM (30m resolution) → 12 hillslopes: width function, slope.
- No routing (travel time channels \ll hillslopes).
- Uniform D, k, f .



Simplification of the hsB equation

- Linearization: the ratio S/w is replaced by $S_c/w=pfD$

$$f \frac{\partial S}{\partial t} = kpD \cos \alpha \left[\frac{\partial^2 S}{\partial x^2} - \left(\frac{1}{w} \frac{\partial w}{\partial x} \right) \frac{\partial S}{\partial x} - \frac{\partial}{\partial x} \left(\frac{1}{w} \frac{\partial w}{\partial x} \right) S \right] + k \sin \alpha \frac{\partial S}{\partial x} + fNw$$

a

- Simplification: the hillslope width function is assumed to be exponential $w(x) = ce^{ax}$

$$\frac{\partial S}{\partial t} = K \frac{\partial^2 S}{\partial x^2} + U \frac{\partial S}{\partial x} + Nce^{ax}$$

$$K = \frac{kpD \cos \alpha}{f}$$

$$U = \frac{k \sin \alpha - akpD \cos \alpha}{f}$$

Validity domain:

-hsB equation: thin conductive soil, flow parallel to an impervious bedrock, uniform k and f , and negligible influence of unsaturated zone.

-Linearization : constant slope and S profile close to average S profile.

Characteristic response function

Analytical approach to establish explicit links between the hydrological response of the hillslope and its geometric and hydraulic properties.
(Brutsaert, 1994, WRR, unconfined aquifer)

CRF = hydrograph normalized by total flow volume [time⁻¹] during drainage of the hillslope.

Spatial dimension of the system → CRF influenced by initial and boundary conditions.

Partial differential equation becomes an ODE in the Laplace domain:

$$K \frac{\partial^2 \tilde{S}}{\partial x^2} + U \frac{\partial \tilde{S}}{\partial x} - s\tilde{S} = -S_0$$

Dimensional analysis

Dimensional analysis to find hydrological similarity

parameters:

$$\frac{\partial^2 \tilde{S}^*}{\partial x^{*2}} + Pe \frac{\partial \tilde{S}^*}{\partial x^*} - s^* S^* = -S_0^*$$

$$Pe = \frac{UL}{2K} = \frac{L}{2pD} \tan \alpha - \frac{aL}{2}$$

Hillslope Peclet number:

$Pe \ll 1$ -> diffusion dominates.

$Pe \gg 1$ -> advection dominates.

Pe only depends on the geometry of the hillslope.

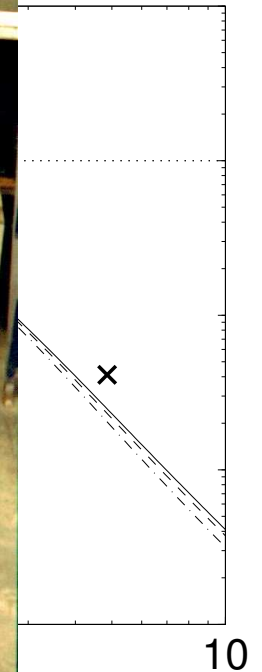
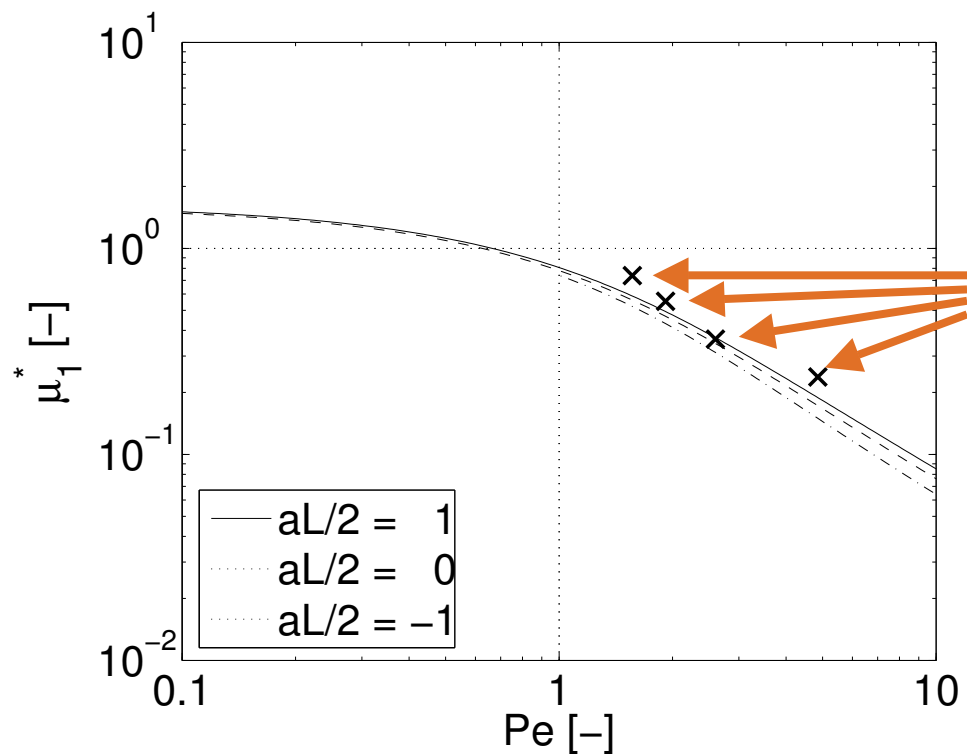
Laplace transform of CRF = moment generating function
→ analysis of the CRF through its dimensionless moments:

$$\tau_K = L^2/(4K) \quad m_n^* = f_n(Pe, \pi^*) \quad \mu_n^* = \sum_{k=0}^n (-1)^{n-k} C_n^k m_k^* m_1^{*n-k}$$

π^* represents the dimensionless parameters related to initial and boundary conditions.

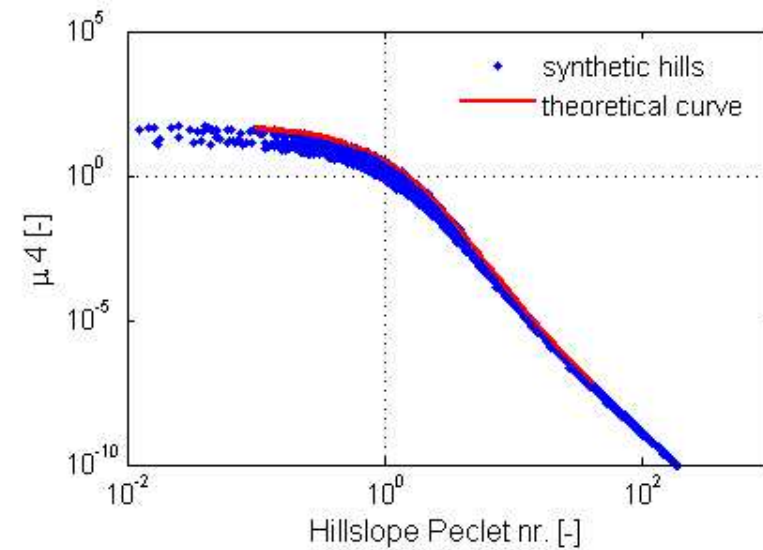
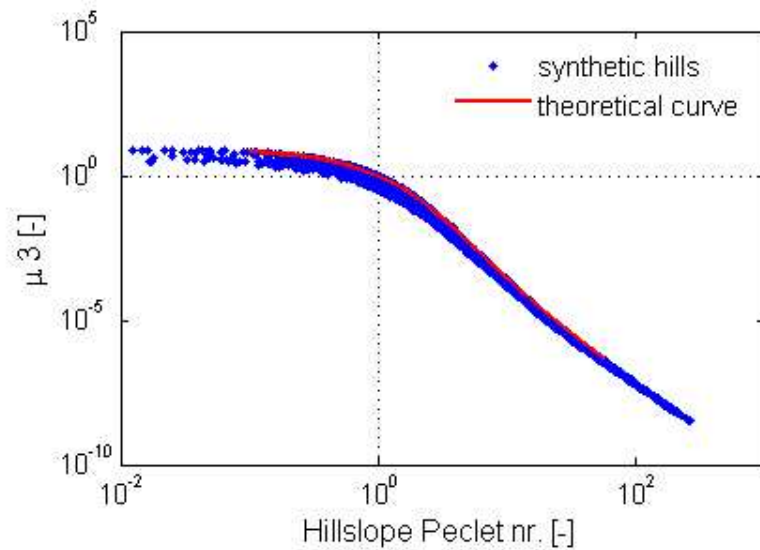
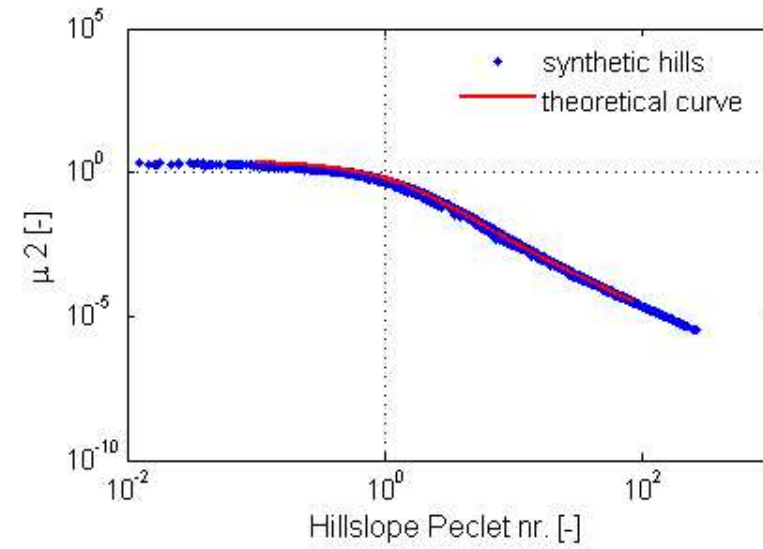
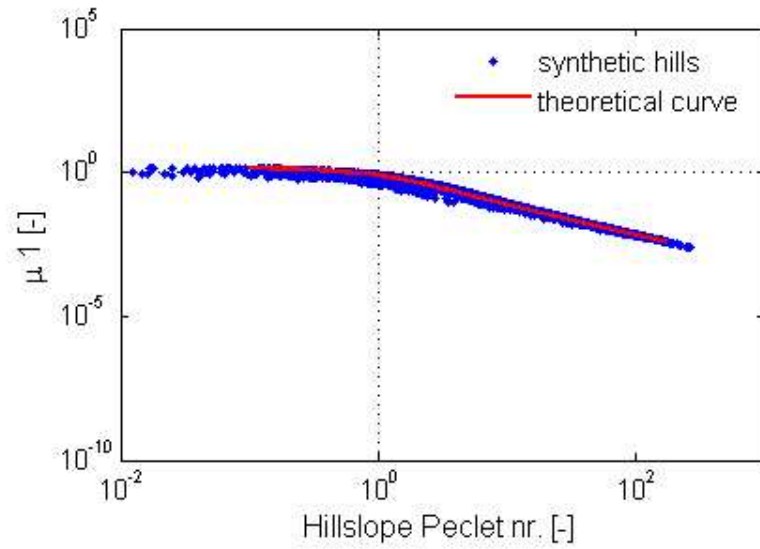
1st and 2nd dimensionless CRF moments

μ_1^* (mean response time)



- Limited influence of
- Good agreement with experimental

Analytical (linear) vs numerical (non-linear) dimensionless moments



Conclusions

Analytical solution to linearized hsB equation provides useful insights into the hydrological behaviour of basic hillslopes:

- Pe efficient similarity parameter for the dimensionless CRF moments (at least order of magnitude).
- Pe only depends on the geometric properties of the hillslope.
- Hydraulic properties define the absolute values of CRF moments.
- Initial testing with data shows promising results.

Perspectives

- Confront with various experimental data to assess validity.
- Similar approach at the catchment scale.
- Potential for ungauged basin applications.
- Development of hsB: variable slope, unsaturated zone, solute transport.
- Rainfall-induced shallow landslides.
- Links between landscape properties and hydrological behaviour.